# Robustness of structures – a theoretical framework

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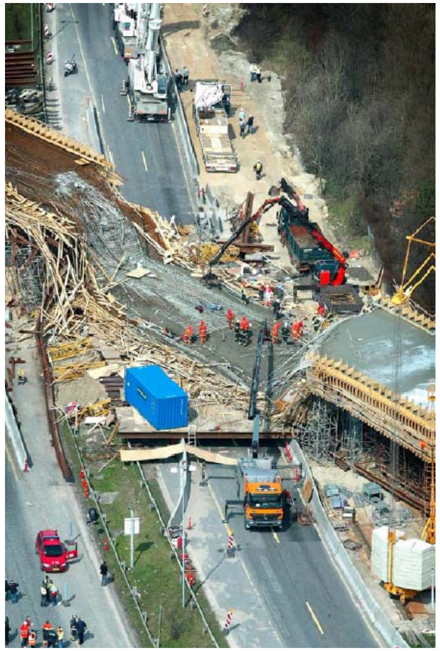
- Introduction
- Framework for robustness assessment of structures
  - Reliability-based approach
  - Risk-based approach
  - Robustness indicators
- Robustness strategies
- Concluding remarks

# Robustness of structures – a theoretical framework

Reasons to failures:

- Extreme high load / extreme low strength: very unlikely (probability of failure per year ~ $10^{-5}$ - $10^{-6}$ )

  covered by ‘component-based’ design rules and psf in codes
- Other reasons:
  - Design errors
  - Execution errors
  - Deterioration of critical structural elements / lack of maintenance
  - Unexpected hazards - unforeseeable incidents
  - System effects



→(to be) covered (partly) by ‘Robustness requirements’ in codes

# Robustness – Theoretical framework

Ballerup arena - 2003

Copenhagen, Denmark

2 out of 12 main trusses collapsed

Ice skating arena - 2006

Bad Reichenhall, Germany

Total collapse

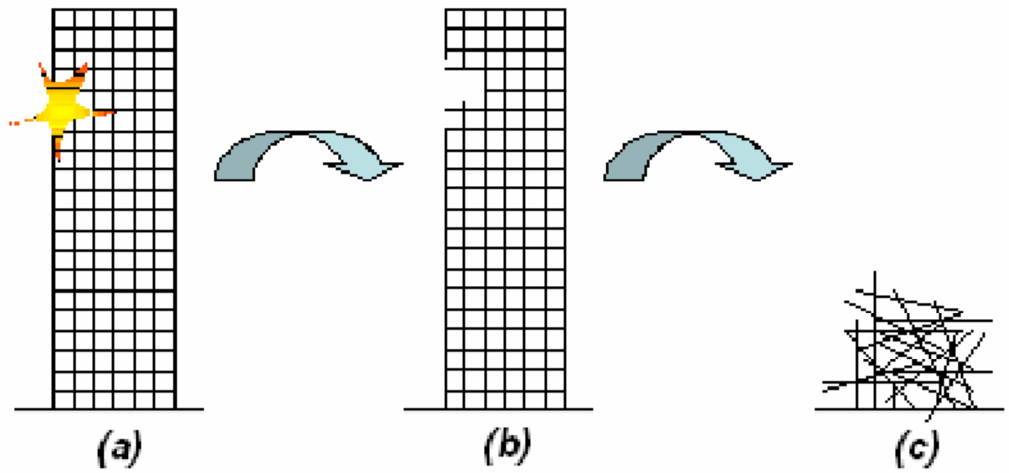
- Hazards: design error, unforeseen incidents, …
  - Correlated / uncorrelated for different elements?
  - New / conventional system?
- Connection between main trusses/beams: strong / weak?
  - Series / parallel (redundant) system?
- Brittle / ductile failure type?

# Robustness - Eurocodes

EN1990 and EN1991-1-7

A structure shall be designed and executed in such a way that it will not be damaged by events such as :

- explosion,
- impact, and
- the consequences of human errors, to an extent disproportionate to the original cause.

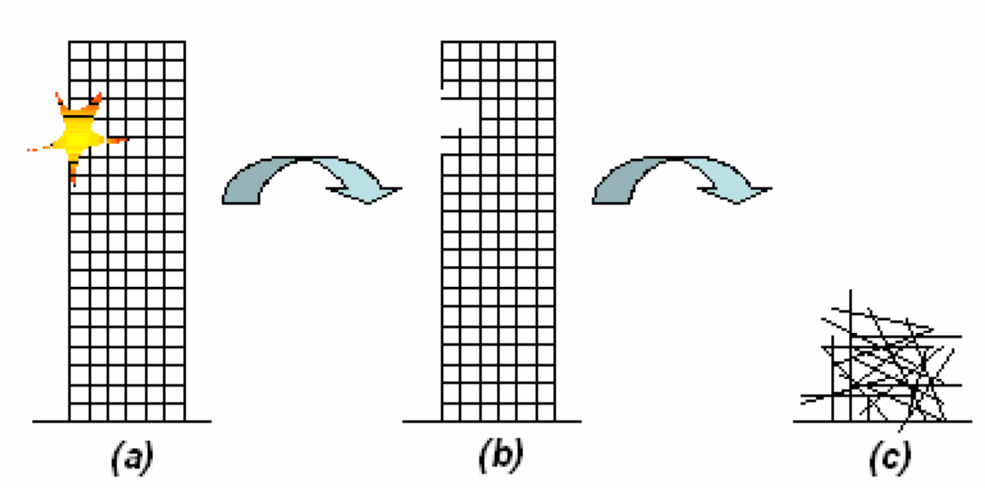


# Robustness - Eurocodes

Potential damage shall be avoided or limited by:

- avoiding, eliminating or reducing the hazards to which the structure can be subjected
- selecting a structural form which has low sensitivity to the hazards considered
- selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localised damage
- avoiding as far as possible structural systems that can collapse without warning → *(ductility)*
- tying the structural members together

# Robustness – probabilistic model

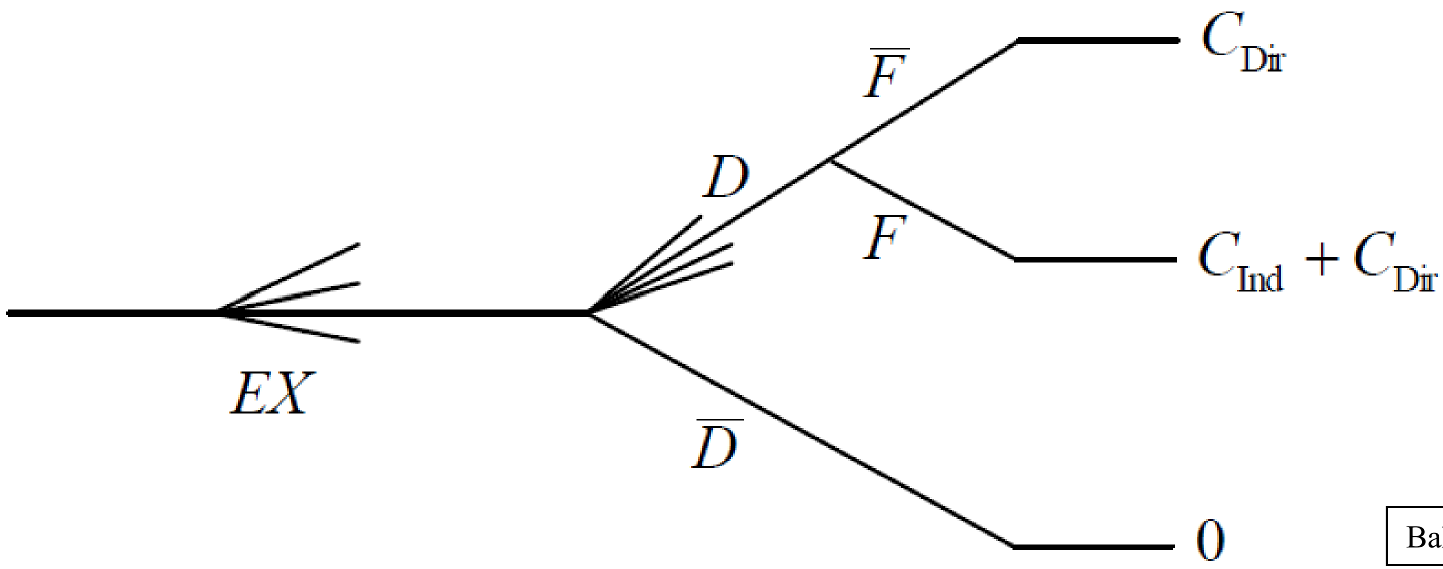



- Exposure - $EX_i$: $\quad P(EX_i)$
- Damage due to exposure - $D_j$ : $\quad P(D_j | EX_i)$
- Consequence – Collapse: $\quad P(\text{Collapse} | EX_i \cap D_j)$

Total probability of collapse:

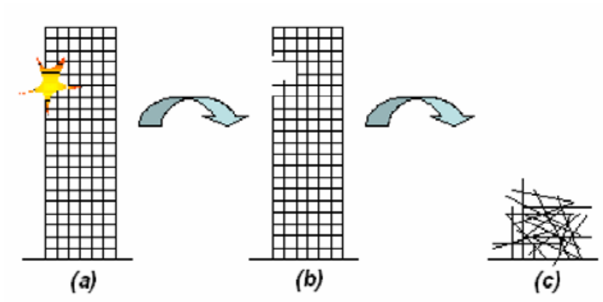
$$P(\text{Collapse}) = \sum_i \sum_j P(\text{Collapse} | EX_i \cap D_j) P(D_j | EX_i) P(EX_i)$$

# Robustness – risk-based model




Baker et al. 2008

Total Risk = Direct Risk + Indirect Risk:



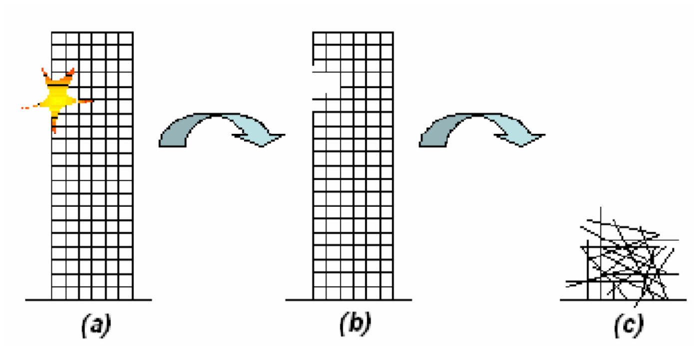

$$R = \sum_i \sum_j C_{\mathrm{dir},ij} P\left(D_j \middle| EX_i\right) P\left(EX_i\right) +$$

$$\sum_k \sum_i \sum_j C_{\mathrm{ind},ijk} P\left(S_k \middle| D_j \cap EX_i\right) P\left(D_j \middle| EX_i\right) P\left(EX_i\right)$$

# Robustness - Theoretical framework

| | prevent disproportionate **failure spreading** | prevent **failure initiation** | |
|---|---|---|---|
| | by controlling global **system behaviour** | by controlling local **component behaviour** | by controlling **abnormal events** |
| $P[C]$ = | $P[C \mid D]$ • | $P[D \mid E]$ • | $P[E]$ |
| | **damage tolerance** | **vulnerability** | **exposure** |
| | **robustness** | | |

Starossek and Haberland 2010



# Robustness - Indicators

- Risk-based robustness index

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}}$$

high robustness : $I_{rob} \to 1$

low robustness : $I_{rob} \to 0$

- Reliability-based robustness indices

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}}$$

high robustness : $\beta_R \to \infty$

low robustness : $\beta_R \to 0$

$$RI = \frac{P_{f\,(\text{damaged})} - P_{f\,(\text{intact})}}{P_{f\,(\text{intact})}}$$

high robustness : $RI \to 0$

low robustness : $RI \to \infty$

- Deterministic robustness index, e.g. based on a pushover analysis

$$RIF_i = \frac{RSR_{\text{damaged}}}{RSR_{\text{intact}}}$$

high robustness : $R_i \to 1$

low robustness : $R_i \to 0$

# Robustness - Indicators

Conditional risk-based robustness indicator:

$$I_{rob|\text{exposure/damage}} = \frac{R_{Dir|\text{exposure/damage}}}{R_{Dir|\text{exposure/damage}} + R_{Ind|\text{exposure/damage}}}$$

- conditional on given exposure and/or damage

# Robustness

How to decrease risk / increase robustness?

$$\text{Risk} = \sum_i \sum_j C_{\text{dir},ij} P(D_j | EX_i) P(EX_i) + \sum_k \sum_i \sum_j C_{\text{ind},ijk} P(S_k | D_j \cap EX_i) P(D_j | EX_i) P(EX_i)$$

Reduce exposures
- Action / hazard models

Limit local damages
-Component models

Reduce extensive damages / collapse
-Post failure models
-Redundancy, ductility, …
-Reduce / strengthen key elements

Reduce direct / indirect consequences

# Robustness

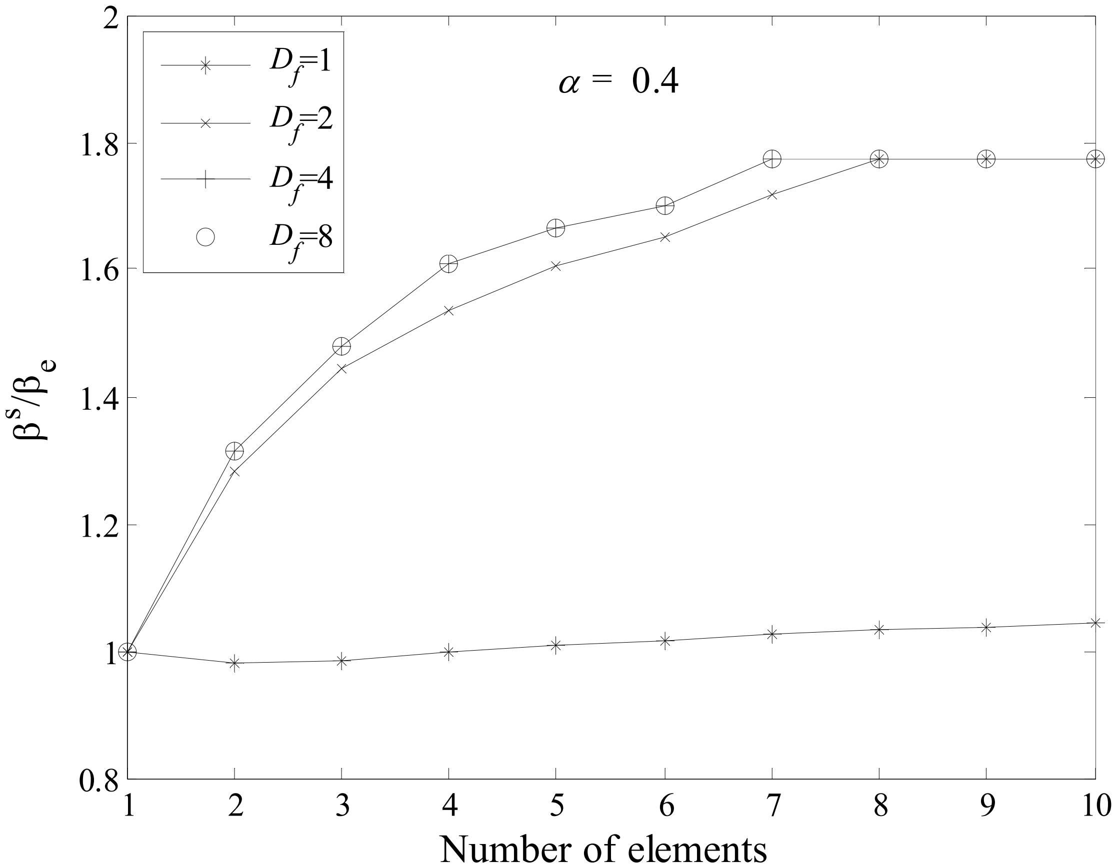
Potential damage shall be avoided or limited by:

- Avoiding, eliminating or reducing hazards
- Structural design with low sensitivity to hazards
- Structural design that can survive adequately the accidental removal of an individual member or limited part of the structure
- Avoiding structural systems that can collapse without warning → (ductility)
- Tying structural members together

- Requirements depend on consequence class (CC1, CC2 or CC3)

# System effects - ductility

Parallel system with ductile elements

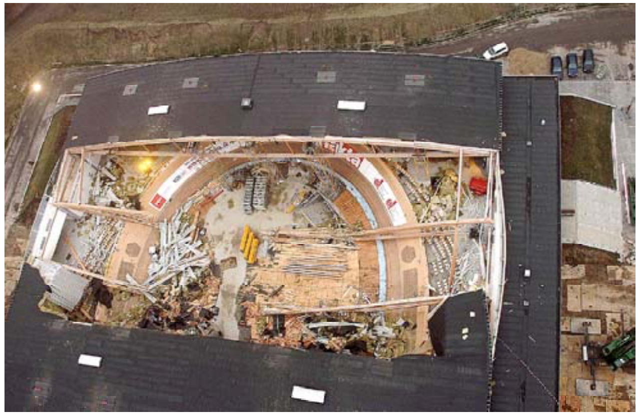


# Robustness strategies

Robustness strategy depend on

- Exposure type: design error, unforeseen incidents, …
- Correlation of exposure between elements
- New / conventional structural system
- Load bearing capacity: time dependency
- Load type: permanent / variable load dominating

# Robustness strategies



- Local effects / exposures - e.g. Local overloading from e.g. local snow accumulation

  → Robustness Approach: Redistribution of loads - e.g. redundant secondary system

- Global effects / exposures – e.g. Global weakening of structural elements due to systematic mistakes

  → Robustness Approaches:

  Large-span structures: Compartmentalization / Segmentation?

  High-rise structures: Redundant, ductile system?

# Concluding remarks

- Reliability- and risk-based basis for assessment of robustness is available

Next steps

- Dissimilation to ‘code committees’ and ‘practicing engineers’
  - Theoretical framework on structural robustness
  - Robust structural design for practising engineers
  - COST E55: Guideline - Design for Robustness of Timber Structures
- Implementation
  - Updating of Eurocodes EN1990 and EN1991-1-7
    TC250 – ‘WG6 Robustness’ and ‘EN 1990 Expert group’
  - Updating of JCSS Probabilistic Model Code
  - …